AN ELECTRONIC PHASE-SEQUENCE AND POWER-FACTOR METER

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ABSTRACT. A Phase-Sequence Meter which can be used to identify the leading and the lagging phases, or to find the nature of the phase-shift produced by a network is described. The technique of measurement is simple and the frequency range of the instrument is quite large. The identification is absolutely dependable when the phase difference is not close to 0 or π. The instrument can also be used as a power-factor meter.

INTRODUCTION

The measurement of phase difference between two voltages of the same frequency are often needed in Electrical and Electronic Engineering Laboratories. Various sophisticated methods are available to Radio and Power Engineers, but generally equipment needed is quite exponsive.

Quite frequently, however, in addition to knowing the phase difference between two phases, one is faced with the problem of identifying the leading and lagging phases. The determination of phase sequence of three-phase or poly-phase systems and finding out the nature of the phase-shift produced by a network, amplifier or a transformer at different frequencies are of great importance in Physical and Engineering laboratories. The instrument described can be readily assembled for the above purposes.

The limitations of the instrument are described later.

PRINCIPLE OF THE PHASE-SEQUENCE METER AND ITS CONSTRUCTION

The schematic diagram of the P.S.M. is given in figure 1. In its simplest form it consists of two cathode-coupled amplifier-tubes A_1 and A_2 having a common load resistance R_L. The voltages whose phase difference and phase sequence are to be determined, are supplied to the inputs of the amplifier tubes through suitable potentiometers or attenuators (P_1 and P_2).

An adjustable Phase-Shifter (P.Sh.) is interposed between potentiometer P_2 and the amplifier A_2 so that the nature and amount of the phase-shift introduced is known. (An R–C phase-shifter is shown in figure 1.) In most of the cases very accurate magnitude of this phase-shift need not be known. A simple phase-shifter can therefore be used. If a continuously variable phase-shifter is not available, one with limited range, can also be employed.

The method of the identification of the voltage (leading and lagging) consists in measuring the relative amplitudes of the voltages individually, and then measuring the amplitude when they are superposed. A known phase-shift is introduced to one of the voltages and the process is repeated. The ratio of the relative amplitude in the second case with respect to that in the first case (K), directly identifies the leading or the lagging voltage, in most of the cases. The conditions of absolute identification are mentioned.

The details involved in carrying out this principle can take a variety of forms. Two simple methods will be given here.

In figure 1, a double-triode has been used as the two cathode-coupled amplifiers A_1 and A_2. A high μ double-triode is preferred to two individual tubes, as it is generally found that the two halves of the double-triode are more identical than two triodes.

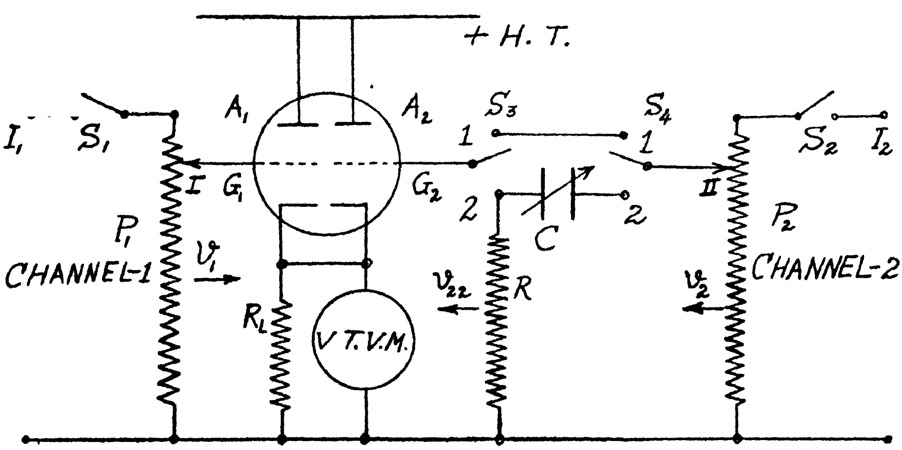

Figure 1. Schematic diagram showing the P.S.M.

Let the two voltages applied to the input terminals I_1 and I_2 respectively be $V_1 \sin \omega t$ and $V_2 \sin (\omega t + \theta)$. With the help of the potentiodmeters P_1 and P_2 these voltages are reduced to ($|v_1| = |v_2|$) equality at the points I and II. It is assumed that no phase changes have been introduced by the potentiometers. This assumption is valid for audio and radio frequencies upto 50 M.c.s. or above. For still higher frequencies the potentiometers are to be replaced by capacity-compensated voltage-dividers. Assuming that the P.Sh. is at zero setting or is not included at all, the voltage developed across R_L due to v_1 at the input terminal I is given by

$$v_1' = \frac{\mu_1}{\mu_1+1} \cdot \frac{Z_L \cdot v_1}{\frac{R_{P1}}{\mu_1+1}+Z_L} \qquad \dots \quad (1)$$

where μ_1 : the amplification factor.

Z_L : is the load impedance of A_1 and A_2, and is the combination of R_L and the shunt input capacitance of the vacuum tube voltmeter, V.T.V.M.

R_{P1} = is the slope resistance of the tube at the operating conditions.

v_1' = the reading of the valve-voltmeter, when S_1 is closed, but S_2 open.

If the amplifiers A_1 and A_2 behave similarly then $\mu_1 = \mu_2$ and $R_{P1} = R_{P2}$ and therefore, a similar equation as equation I will give the voltage developed across Z_L due to the voltage at the input point II. The two voltages at points I and II are equal, as already stated, and each is equal to v_1. This shows that the relative phase difference between the two voltages v_1' and v_2' will be the same as that between original input voltage V_1 and V_2. It also shows that if μ is high and Z_L is not very low, then even if the two amplifier-sections do not behave exactly identical (due to small difference in the values of R_P and μ), the magnitudes of v'_1 and v'_2 will be equal.

It is possible to measure the phase difference between v_1 and v_2 by taking the valve-voltmeter (V.T.V.M.) reading when both the voltages are simultaneously applied to A_1 and A_2, and then taking the readings when one switch or the other of S_2 and S_1 are kept open. The readings correspond respectively to V_{R1}, v'_1 and v'_2 respectively.

Now,

$$\begin{aligned} V_{R1} &= \bar{v}'_1+\bar{v}'_2 = (v_1'^2+v_2'^2+2v_1'v_2' \cos\theta)^{\frac{1}{2}} \\ &= \{2v_1'^2(1+\cos\theta)\}^{\frac{1}{2}} \\ &= 2v'_1 \operatorname{Cos}\theta/2 \qquad \dots \quad (2a) \end{aligned}$$

Therefore relative amplitude of the resultant with respect to one of the components is given by,

$$(R.A.)_1 = \frac{2v'_1 \operatorname{Cos}\theta/2}{v_1'} = 2 \operatorname{Cos}\theta/2 \qquad (2b)$$

If a known phase-shift (ϕ), is now introduced to one of the two input voltages and their amplitudes are again adjusted to equality, then with a prior knowledge of the $\theta/2$ or θ from the equation (2*b*), it is possible to find out which is leading or lagging phase. In some cases, where ϕ should be exactly known, it can either be calculated out, or can be measured with the help of P.S.M. itself, provided a calibrated phase-shifter is not available.

DETECTION OF THE LEADING OR LAGGING VOLTAGE

The Author has used a simple R–C phase-shifter. The output voltage v_{22} across the resistance arm R leads the input voltage v_2 by an angle ϕ (figure 1).

where

$$\phi = \tan^{-1} \frac{\quad}{\omega CR} \tag{3}$$

and

$$\mathrm{Cos}\ \phi = \ v_{22} \ — \ v'_{22} \tag{4}$$

v'_2 and v'_{22} being the v.t.v.m. readings corresponding to v_2 and v_{22}. The eq. 3 shows that ϕ lies between 0 and $+\pi/2$.

By changing R or C or both, the value of ϕ can be adjusted so as to obtain the maximum sensitivity. (The extreme values of R or C may not be possible to use due to other considerations).

On introducing the phase-shift to the leading phase $v_2 \angle \theta$, this phase-vector rotates through a positive angle ϕ and its magnitude also reduces to v_{22}. With the help of the potentiometer P_1 the input to the other channel (v_1) is reduded to v_{11}, which is equal to v_{22}. This is indicated by the corresponding v.t.v.m. readings $v'_{11} = v'_{22}$.

The relative amplitude $(R.A.)_2$ and the ratio K can now be written as

$$(R.A.)_2 = 2\ \mathrm{Cos}\ (\theta+\phi)/2 \quad \ldots \tag{5}$$

and

$$K = \frac{(R.A.)_2}{(R.A.)_1} = \frac{\mathrm{Cos}(\theta+\phi)/2}{\mathrm{Cos}\ \theta/2} \tag{6}$$

It is clear that value of θ greater than π need not be considered, as in these cases the leading phase behaves as the lagging phase and *vice versa*.

Remembering that ϕ is less than $\pi/2$ and positive, if the values of θ and ϕ are systematically arranged, we arrive at the following results given below in table 1.

Table 1

(I) If ϕ is applied to the *leading phase*
and if $(\theta+\phi) < \pi$ then $K < 1$, i.e., $(R.A.)$ decreases

(II) If ϕ is applied to the *lagging phase*
and if $(\theta-\phi) > 0$ then $K > 1$, i.e., $(R.A.)$ increases.

These two opposite effects, namely the decrease or increase of the *R.A.* make the identification of the leading and the lagging voltages a certainty under the conditions mentioned. As ϕ is adjustable, simultaneous satisfaction of the conditions $(\theta+\phi) < \pi$, and $(\theta-\phi) > 0$, is a simple affair, unless θ is very close to 0 or π. A small ϕ automatically satisfies the above conditions, and need not be actually calculated. Replacing the simple R–C phase-shifter by a constant amplitude one, it is possible to make the measurements simpler. The *R.A.* can be measured before and after the phase-shift, and their ratio can be directly calculated without having to adjust the potentiometer P_1 for equalizing the amplitudes after the phase-shift. The ratio K will be given by the same formula (eq. 6) as before, and the method of identification will remain the same.

EXPERIMENTAL

The schematic diagram of the single tube phase-sequence meter assembled by the author is shown in figure 1.

Details of the different steps for detection of the phase-sequence are given in short in the following section :—

(1) The switches S_3 and S_4 are thrown to the poles 1, 1, initially.

(2) The switches S_1 and S_2 are closed one at a time, and the readings of v.t.v.m. are adjusted to equality ($|v'_1| = |v'_2|$) with the help of P_1 and

(3) S_1 and S_2 are closed together and the v.t.v.m. reading corresponding to the combined voltage is taken.

(4) From the above measurement $(R.A.)_1$ and θ are calculated.

(5) The phase-shifter (R–C network) is now introduced by throwing S_3 and S_4 to the poles 2, 2. S_1 is opened. The v.t.v.m. reading gives v'_{22} and therefore ϕ (eq. 4). By adjusting C, ϕ is made less than θ, and at the same time $(\theta+\phi)$ less than π.

(6) S_2 is opened and S_1 closed, P_1 is adjusted until v'_{11} becomes equal to v'_{22}, and $(R.A.)_2$ is found as before (step-3).

(7) The phase-identification is done by consulting the table 1, and noting the increase or decrease of *R.A.*

(8) Inputs to the channels are reversed, and the observations are repeated for double verification.

An alternative arrangement : (Write *et al*, 1936; Ragazzini, 1950).

The measurement of θ and ϕ are affected at high frequency due to stray reactances present with unequal potentiometer sections. As the identification of phases is dependent on the measurement of the above angles, it is advisable to use equal potentiometer sections and to feed unequal voltages to the input channels of the P.S.M.

A constant-amplitude phase-shifter (Alton Everest, 1941) is used in this case, instead of a R–C phase-shifter.

In this case θ is given by the eq.

$$E_1 = (v_1'^2 + v_2'^2 + 2v'_1 v'_2 \operatorname{Cos} \theta)^{\frac{1}{2}}$$

where v'_1 and v'_2 have the same significance as before, and E_1 is the reading of the v.t.v.m. when both the voltages are combined.

similarly,

$$E_2 = \{v_1'^2 + v_2'^2 + 2v'_1 v'_2 \operatorname{Cos} (\theta \pm \phi)\}^{\frac{1}{2}}$$

where E_2 is the superposed reading, after the phase-shift. The positive or negative sign has to be used accordingly as ϕ has been applied to the leading or lagging phase.

As before we find

$$K = \frac{E_2}{E_1} < 1,$$

when $(\theta + \phi) < \pi$, and ϕ is applied to the leading phase.
and

$$K = \frac{E_2}{E_1} > 1,$$

when $(\theta - \phi) > 0$, and ϕ is applied to the lagging phase.

These results are same as those in table 1 and can be taken as general.

Measurement of ϕ.

In the alternative arrangement, ϕ can be directly read or calculated, but the author has used the P.S.M. itself to measure ϕ, as it is most dependable. figure 2 shows the switching arrangement for this. A brief statement of the various steps followed is given below :—

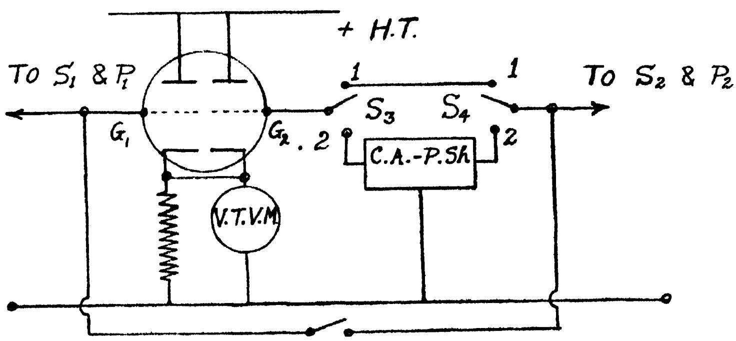

Figure 2. Measurement of ϕ by the P.S.M. The Phase Shifter is a constant amplitude one.

S_1 open, S_2 closed, S_3 and S_4 at poles 2, 2, S_5 closed, v_2 applied to the channel 2,—then at G_1 is impressed the voltage v_2 and at G_2 the same voltage v_2 with rotation ϕ.

ϕ is given by the equation (2*a*.) which can be written as

$$V' = 2v'_2 \operatorname{Cos} \phi/2$$

where V' is the v.t.v.m. reading when both the voltages are superposed and v_2' has same meaning as before. The rest of the measurement is similar to that in the previous section and can be done after opening switch S_5.

In case of the R–C phase-shifter ϕ can be measured by taking the ratio $\left|\frac{v'_{22}}{v_2'}\right|$, (equation 4).

DISCUSSION

Effect of frequency: This has already been mentioned in the "alternative method". The effect of equal reactances in the two channels are identical and hence θ remains unaltered.

Advantages: The arrangement of the P.S.M. is somewhat differential in nature, and therefore any common H.T. variation does not affect the measurement. A high-μ tube is preferred as the difference in the values of the parameters produce least effect on the amplifications.

The possibility of using a phase-shifter of a limited range, and that without the need of knowing the phase-shift accurately in most of the cases, is a great advantage over the common-plate-load-type (C.P.L.T.) phase-meters. A simpler network with higher accuracy and lower cost can therefore be employed.

The potentiometer-method of amplitude control is superior to the conventional method of gain control by varying the transconductance of one of the tubes in the C.P.L.T. instruments. If the cathode-bias is varied for this purpose, the phase-shift produced by the two amplifiers may not be identical.

With all other usual advantages of the cathode-follower arrangement, an excellent frequency response also exists.

Limitations: It is evident that the sensitivity of measurement cannot be same at all the ranges, as the value of K will depend on $\operatorname{Cos} \theta/2$ and $\operatorname{Cos} (\theta+\phi)/2$ and the rate of variation of these quantitatics are not uniform over all the values. Quite often different ranges of the v.t.v.m. have to be used. However, ϕ can be adjusted to give the maximum sensitivity for a particular value of θ.

APPLICATIONS

The P.S.M. with certain modifications can be conveniently used for the following purposes:

(A) to measure the Power Factor (P.F.) of a single-phase or three-phase balanced loads

(B) as a phase-sequence-meter (P.S.M.) for poly-phase systems.

(A) In the case of single phase circuits the procedure is quite simple. A non-inductive resistance R_1 is included in series with the lines as shown in figure 3.

Feeding the corresponding input points of the P.S.M. from the points *a,b* and *b,c* respectively, the phase difference and the P.F. can be determined as usual. The voltage across R_1 is proportional to the line current and the voltage across the load is given between the pts. *c,b*. R_1 may be a variable resistance so that the equal potentiometer sections could give equal input voltages. The Potentiometers should have comparatively high values, so as not to modify P.F. of the load.

The angle of P.S.M. is physically measuring is $\psi = \pi - \theta$ and not θ. This occurs due to the fact that one of the voltages E_{bc} has been fed in the reversed phase into the channel-2. As Cos $\theta = -$Cos ψ, the P.F. remains the same, neglecting the change of sign (figure 3)

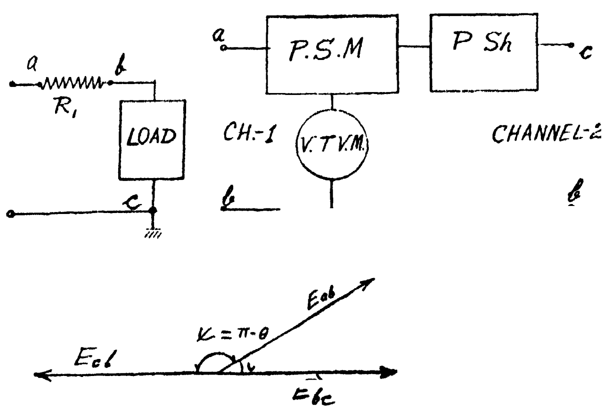

Figure 3. Measurement of Power Factor (single-phase). As '*b*' is the common reference-point, the voltage E_{bc} is fed in the reversed phase into the channel-2.

This phase reversal effectively changes the leading voltage into the lagging voltage, and *vice versa*; while determining the nature of the load this must be kept in mind.

In the case of three-phase, three-wire system (either star or delta) with a balanced load, a line voltage vector (V_{1-2}) leads the corresponding line current vector I_1 by an angle $30 \pm \theta$, where θ is the phase difference between the emf and the current in each phase. The plus or minus sign is employed accordingly as the load is inductive or capacitative.

A non-inductive resistance R_1 is included in one of the lines, say 1, and the voltage across it (R_1 I_1) is fed into the channel 1 of the P.S.M. The voltage V_{1-2} between the lines 1 and 2, is fed into the other channel. The angle the P.S.M. measures is given by $\psi = \pi-(30\pm\theta) = 150\mp\theta$. This is because the voltage V_{1-2} is fed with a phase-reversal. (figure 4).

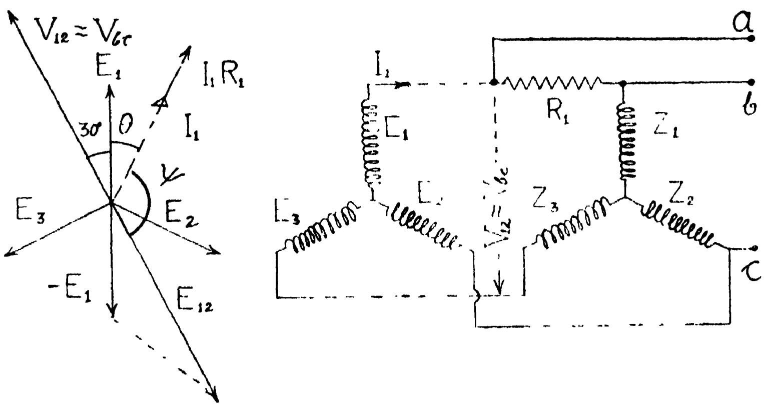

Figure 4. Measurement of Power Factor of 3-phase balanced load (3-wire, symmetrical system). The vector diagram shows the angle ψ.

It is also implied above that the phase-sequence and the identity of I_1 and V_{1-2} are already known.

(B) The importance of knowing the phase sequence of poly-phase system is well-known. The P.S.M. can be conveniently used for this purpose.

The phase-difference between two successive phase or line voltages in a symmetrical system is given by $2\pi/n$, where 'n' is the number of phases which seldom exceeds six. Hence the conditions $(\theta+\phi) < \pi$ and $(\theta-\phi) > 0$, can be easily attained.

With a prior knowledge of 'n' it is possible to pick up any two voltages arbitrarily and to determine their phase-difference as well as phase-sequence relative to each other. As the phase-difference is an integral multiple of $2\pi/n$ in the case of a balanced system, the relative position of the phase-vectors will be known. A third voltage can be paired with any of the two above and the process repeated.

In case of three-phase system only two readings are needed to find the phase-sequence.

In an unbalanced system phase-sequence can be determined as before. It should be however be remembered that the phase-vectors are not symmetrically oriented in this case.

The determination of the phase-sequence is of great importance to the Power and Electronic Engineers and the above instrument offers not only a very simple method for that purpose, but the phase angle between two voltages and consequently the power-factor can also be determined.

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(a) The base-line '*bb*' of the P.S.M. should not be grounded, as one of the supply lines in a single-phase system and the star-point in a 3-phase system is normally grounded (fig. 3).

(b) Apparent involved nature of the phase relations in a poly-phase system has nothing to do with the P.S.M.,—it is inherent in the system itself.

(c) It is assumed that R_1 is sufficiently small resistance compared to the load impedance so as not to modify the balanced condition or symmetrical working of the system. This implies that the voltage drop across R_1 is quite small. The accuracy of θ will depend on the validity of the above condition.

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